LECTURE-1

We have,

$\frac{dy}{dx}$ → Means derivative of y w.r.to x

y → Dependent variable

x → Independent variable

NOTATIONS

$$y' = \frac{dy}{dx}, \quad y'' = \frac{d^2y}{dx^2}, \quad y''' = \frac{d^3y}{dx^3} \quad \text{etc.}$$

What is a differential equation?

It is simply the relation between the derivatives.

Example:-

(i) $y'' + 2y' + 3y = x^2$

(ii) $y' - 4y = 0$

TYPES OF DIFFERENTIAL EQUATIONS

There are two types of differential equations

(i) Ordinary differential equation (ODE)

(ii) Partial differential equation (PDE)

ODE

A differential equation involving only one independent variable is known as an ODE. General derivative is used there.

Example:

$$\frac{dy}{dx} - 3y = 0 \quad \text{or} \quad y' - 3y = 0$$

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - y = 0 \quad \text{or} \quad y'' + 4y' - y = 0$$

PDE

A differential equation involving more than one independent variable is known as a PDE. Partial derivative is used there.

Example:-

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad \text{or} \quad u_x + u_y = 0$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \quad \text{or} \quad u_{xx} + u_{yy} + u_{zz} = 0$$

ORDER AND DEGREE

A differential equation is said to be of order n if the highest order derivative is n.

Degree is the power of the highest order derivative.

Example:-

$$\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^3 + y = 0$$

order – 2, degree-1

Example:-

$$\frac{d^3y}{dx^3} = \sqrt{\frac{d^4y}{dx^4}}$$

Squaring both sides.

$$\left(\frac{d^3y}{dx^3}\right)^2 = \frac{d^4y}{dx^4}$$

order – 4, degree-1

Example:-

$$\sqrt{\frac{d^3y}{dx^3}} = \sqrt[3]{\frac{dy}{dx}}$$

Taking the sixth power of both sides,

$$\left(\frac{d^3y}{dx^3}\right)^3 = \left(\frac{dy}{dx}\right)^2$$

order - 3, degree - 3

Diffrent kinds of solutions

Ex:-1

Solve, $\frac{dy}{dx} = 1$

Ans:- We have, $\frac{dy}{dx} = 1$

$$\Rightarrow dy = 1\,dx$$

Integrating both sides.

$$\int dy = \int 1\,dx$$

$$\Rightarrow y = x + c_1$$

Ex:-2 Here the order of the differential equation is 1. The solution contains one arbitrary constant.

Ex.-2

Solve $\frac{d^2y}{dx^2} = 1$

Ans:- We have, $\frac{d^2y}{dx^2} = 1$

$$\Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = 1$$

$$\Rightarrow d\left(\frac{dy}{dx}\right) = 1\,dx$$

Integrating both sides,

$$\int d\left(\frac{dy}{dx}\right) = \int 1\, dx$$

$$\Rightarrow \frac{dy}{dx} = x + c_1$$

$$\Rightarrow dy = (x + c_1)\, dx$$

Again integrating both sides.

$$\int dy = \int (x + c_1)\, dx$$

$$\Rightarrow y = \frac{x^2}{2} + c_1 x + c_2 .$$

Here the order of the differential equation is 2. The solution contains two arbitrary constants.

NOTE:- Thus the solution of a nth order differential equation contains n arbitrary constants is called a general solution.

Initial condition

Ex:- $\frac{dy}{dx} = 1$

$y(0) = 10$ → initial condition

y ↓ value of the independent variable x

Ex:- $\frac{d^2y}{dx^2} = 1$

$y(2) = 5$ (x, y)

$y'(3) = 11$ (x, y')

Example

Solve $y' = 10$, $y(2) = 3$

Ans: We have, $\frac{dy}{dx} = 10$

$\Rightarrow dy = 10\,dx$

Integrating both sides,

$\int dy = \int 10\,dx$

$\Rightarrow y = 10x + c_1$ ——— (1)

$y(2) = 3$ i.e $y = 3$ when $x = 2$

Putting in equation (1)

$3 = 10 \times 2 + c_1$

$\Rightarrow c_1 = 3 - 20 = -17$

Putting the value of c_1 in equation (1)

$y = 10x - 17$ ——— (2)

Equation (2) is a Particular Solution. which is defined for Particular value of x & y

where as equation (1) is a general solution.

Initial value problem (IVP)

The problem which is defined only for one point of x is known as an IVP

Ex:- $y'' + 2y' + 3y = 0$, $y(0) = 2$, $y'(0) = 5$

Boundary value Problem (BVP)

The Problem which is defined for more than one point of x is known as a BVP.

Ex: $y'' - y' + 5y = 0, \quad y(1) = 4, \quad y'(3) = 10$

Formation of differential equation

Example:-1

Find the differential equation of all straight lines passing through origin.

Ans: We have the equation of all st. lines passing throgh origin is

$$y = mx$$

Diffrentiating both sides w.r to x

$$\boxed{y' = m}$$ is the required differential equation

EX:-2 Find the DE of all circles with centre at $(0,0)$ & radius r.

Ans: The requires eqn of the circle is

$$x^2 + y^2 = r^2$$

Diff. both sides w.r. to x

$$2x + 2yy' = 0$$

$$\Rightarrow \boxed{y' = -\frac{x}{y}}$$ is the required differential equation.